Comparison of Modeling Strategies to Capture Component Level Damage in Non-ductile Reinforced Concrete Buildings

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ABSTRACT

There are many finite element modeling (FEM) techniques used in practice and research to interpret behavior and gather responses of structures subjected to earthquake excitations. The application of particular FEM techniques in a structure typically depends on the engineering demand parameters (EDPs) or structural responses of interest, along with other considerations regarding computational effort and level of fidelity required. A non-ductile reinforced concrete (RC) building may suffer from many design deficiencies when located in a seismically active region. In this paper, a localized shear and joint model are chosen to incorporate into the FEM in order to gather global, intermediate, and local EDPs that correlate with observed areas of concern in non-ductile RC buildings. Four numerical model combinations are developed to perform a probabilistic seismic demand analysis on the gathered EDPs. Based upon the results of these simulations, a new uncertainty value (σ_{FEM}) is established to reflect the uncertainty introduced by adopting different types of numerical models. The σ_{FEM} addresses an uncertainty that exists from assuming more simplified models of structural behavior, and allows researchers and industry to idealize numerical models for analysis while still reflecting this uncertainty.

INTRODUCTION

Significant research has been undertaken to model and predict the performance of non-ductile reinforced concrete (RC) buildings under seismic loading. Ranges of numerical modeling (i.e. Finite Element Modeling) procedures have been developed in research and adopted in practice to assess the vulnerability of such structures subjected to earthquake excitations. In particular, the vulnerability evaluation requires a deep understanding of the nonlinear response of the structure, and hence, modeling the post-elastic behavior of structural components used in dynamic analysis of RC buildings is largely investigated. Based on the level of knowledge required in the structural response, the configuration and the accuracy of the models may differ.

Studies of alternative FEM strategies have typically focused on the comparison of models for an individual component within an RC building (e.g. alternative joint models) (Alath and Kunnath 1995; Altoontash 2004; Aycardi et al. 1994; Elwood and Moehle 2008; Lowes et al. 2004; Mitra and Lowes 2007). Moreover, in most of the cases, the comparison is based on the evaluation of the deterministic behavior rather than by evaluating the seismic response given record-to-record variation among other potential sources of uncertainty. Proper assessment of the seismic vulnerability of structural systems should take into account all pertinent sources of uncertainty, including uncertainties in the seismic input (record-to-record variability), the parameters defining the structure or component limit states (capacity uncertainty) and the properties defining the structural model (model parameter uncertainty). The effects of input and capacity uncertainties have been largely investigated in past studies (Cornell et al. 2002; Vamvatsikos and Allin Cornell 2002). Other researchers have recognized the importance of modeling uncertainties that affect the structural performance of numerical models, including the uncertainties in material and geometric properties and developed methodologies to evaluate their effect on the seismic response (Liel et al. 2009; Padgett and DesRoches 2007; Tubaldi et al. 2012; Vamvatsikos and Fragiadakis 2010). However, limited past research has focused on the effect of using different modeling techniques on the dispersion of the seismic response or vulnerability of RC structures. This topic is gaining importance since a multitude of modeling techniques for RC buildings and their components have been developed in recent years. A new uncertainty parameter accounting for the dispersions from the wide variety of possible FEM modeling techniques should be introduced in the probabilistic assessment of structures.

This paper will address gaps in the probabilistic modeling of non-ductile RC buildings attributed to numerical modeling. The gaps include the use of component models to gather data output for local and intermediate level EDPs and acknowledging a variation of probabilistic assessment results from the FEM combinations. The gaps are addressed by comparing alternative levels of modeling fidelity for non-ductile RC buildings. First their ability to reproduce experimental test data is compared, and subsequently their influence on the probabilistic seismic response evaluation is investigated. Component level responses are emphasized in addition to global responses to highlight the influence of alternative models on probabilistic estimates of local and intermediate EDPs of interest for next generation damage, functionality, and loss assessment. Four different modeled building combinations are considered. The more sophisticated model includes a joint model (Alath and Kunnath 1995) able to capture the joint deformability and a shear model (Elwood 2004) able to reproduce the shear failure, while the simplest model is defined using rigid connections at the joints and is not able to capture shear failure. The deterministic behavior of the models is compared by observing various levels of EDPs. Moreover, the probabilistic behavior of these response parameters for each model is investigated by performing a probabilistic seismic demand analysis (PSDA). Finally, a new FEM uncertainty parameter is developed from dispersion data between the different models to support future damage and risk modeling of RC buildings.

ALTERNATIVE ANALYTICAL MODELING APPROACHES

In this paper a case study structure is adopted to test the influence of alternative modeling strategies, including comparison and validation with past experimental test data of the structure as well as quantification of model influence on probabilistic response assessment at the local and global levels. The case study adopted and the details on the numerical modeling approaches adopted are as follows.

CASE STUDY STRUCTURE

The case study structure adopted is representative of RC buildings typically designed prior to the adoption of modern seismic codes and seismic detailing in Central and Eastern United States (CEUS) as identified by El-Attar et al. (1991). The 3-bay 3-story structure was designed for gravity load only by following the ACI 318-89 producing non-seismic detailing, and was experimentally tested by Bracci et al. (1992a). Material, cross-section details for the columns and beams, as well as distributed loads applied to the structure are reported in Bracci et al. (1992a).

The open-source computational framework OpenSees (McKenna et al. 2006) is adopted to perform nonlinear dynamic analyses. Four different modeled building combinations are considered. The "base model" is denominated as: 1) *rigid model* employing *force based beam-column elements* (McKenna et al. 2006) with fiber discretized cross-sections in order to represent the beams and columns, rigid elements at the joints, and neglects shear deformations. The other models are upgraded by employing local models to represent the behavior of joints and the shear behavior of columns. Hereinafter they are called as: 2) *shear model*, 3) *joint model* and 4) *joint-shear model* and are described in the following sections. The hysteretic behavior of concrete is accurately reproduced by the *Concrete02* uniaxial material (McKenna et al. 2006). The concrete material properties are defined based on an average of the properties as tested from each pour of the one-third scale model constructed by Bracci et al. (1992a). The properties of the confined concrete have been evaluated by using the formulation proposed by Mander et al. (1988) and have been used for the fibers of the core section. However the effect of the confinement has been found to be minimal given the limited transverse reinforcement provided in the structure. The behavior of steel reinforcement is described by the *Hysteretic* uniaxial material (McKenna et al. 2006) whose parameters controlling pinching, damage and degraded unloading stiffness are accurately calibrated. Steel yield strength is increased by 25% from the nominal value of the ASTM 615 Grade 40 steel to provide the model with experimentally tested properties of the steel material (Aslani and Miranda 2005).

RIGID, SHEAR, AND JOINT MODELS

In the structural model termed *rigid model*, the joints are modeled by connecting the adjacent beam and column nodes with idealized perfectly rigid beam-column elements. These rigid links serve to reflect the physical dimension of the joint and allow moments and forces to be fully transmitted through the joint to the surrounding elements. As noted by Celik (2007), potential limitations of such models include their inability to respond to the actual joint panel deformability observed in real structures. Moreover, the joint panel shear has been observed as a critical section of failure in RC buildings (Priestley 1997).

The structural model termed *shear model* adopts a column shear model developed by Elwood and Moehle (2008) along with the previously described fiber modeling of beams and columns for flexural behavior and rigid joints. Elwood and Moehle (2008) proposed the *LimitState* uniaxial material (McKenna et al. 2006) model to be applied to zero length springs along the column. The *LimitState* material acts on the model by updating the column response curve once a shear force limit has been reached. The limits to be reached are defined using a shear limit curve and axial limit curve, which are defined based upon parameters that depend upon the column detailing and orientation used in the case study building. The structure specific parameters integrated in the model include concrete compressive strength, column width, column depth, effective column depth, and transverse reinforcement ratio. The use of this local model permits to account for the instance of shear failure and loss of axial load carrying capacity in the column.

Several researchers have conducted experimental studies and proposed alternative joint models in order to correctly capture the response observed in interior and exterior joints in RC buildings (Alath and Kunnath 1995; Altoontash 2004; Aycardi et al. 1994; Lowes et al. 2004; Mitra and Lowes 2007). Celik (2007) found that a scissors model with rigid end zones best predicted the response of the experimentally tested subassemblages with a reasonable computation effort. A schematic representation of the considered joints is reported in Figure 1.

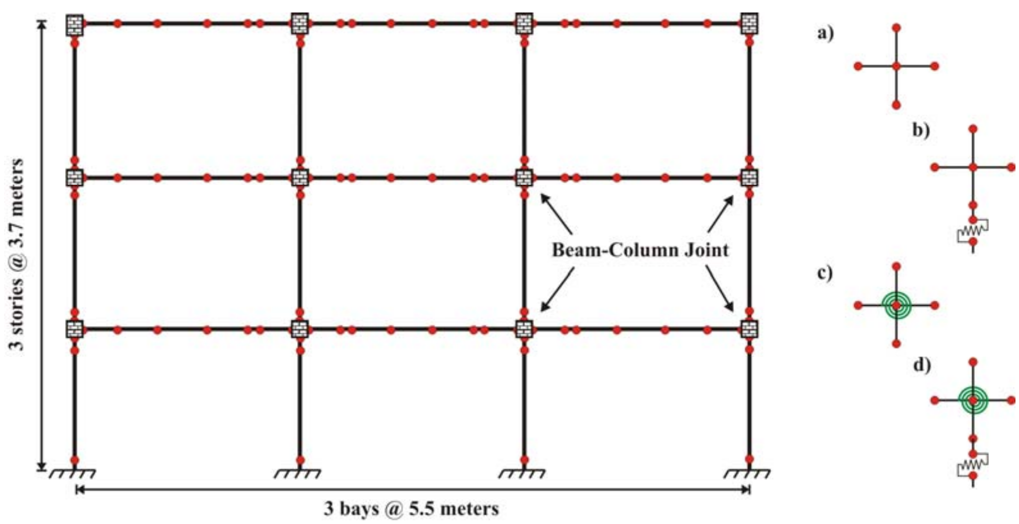

Figure 1: OpenSees two-dimensional model of full scale building with four different combinations of the joint and column spring details: a) rigid, b) shear, c) joint, d) joint-shear. Beam and column section details are found in (Bracci et al. 1992a).

To apply the scissors model in OpenSees, the *Pinching4* uniaxial material (McKenna et al. 2006) is used to describe the rotational behavior of the zero length spring at the joint connection, following the procedure described in further detail in Celik (2007). The *Pinching4* material allows the definition of a multi-linear backbone curve, along with pinched response and strength degradation from unloading and reloading. By incorporating the joint models, the structural model is able to capture the joint panel deformation and degradation of strength during the seismic excitation. The *joint model* utilizes the fiber-discretized beams and columns along with joints modeled using the nonlinear spring as defined herein. The *joint-shear model* utilizes both the aforementioned column shear and joint spring within the same finite element model of the case study building.

MODEL COMPARISONS WITH EXPERIMENTAL DATA

The models described above are compared with available experimental data to gain insight into their relative performance and their ability to reproduce observed behaviors. The lateral load cyclic behaviors of *rigid* and *joint* models are compared with the interior and exterior joint subassemblage experimental results from Aycardi et al. (1994). Also, the dynamic behavior of all the models is compared with the available experimental results from Bracci et al. (1992b) in order to better understand the importance of using local joint and shear models.

SUBASSEMBLAGE MODEL

Aycardi et al. (1994) constructed one-third scale models of interior and exterior subassemblages in order to investigate the experimental response of joints with non-seismic detailing. The subassemblage dimensional specifications and material properties were adopted from the one-third scale 3-story by 3-bay RC frame developed by Bracci et al. (1992a and 1992b). The interior and exterior subassemblages were axially loaded and subjected to reversed cyclic lateral displacements of increasing drifts until failure.

Figure 2 shows the comparisons between the experimental data and the results carried out by the *rigid* and *joint* models for both the interior and exterior subassemblages. From the interior subassemblage comparison, it can be observed that the *joint model* captures the degradation and pinching behavior more accurately as compared to the *rigid model*. While the *rigid model* response compares well with the experimental response in the negative drift range, the response in the positive drift range accounts for only about two-thirds of the experimental lateral load as the subassemblage continues to yield. Despite the improvement in capturing degradation and pinching, the *joint model* under-predicts lateral load in the positive drift range while over-predicting by twenty percent in the negative drift range. Such relative over- and under-predictions, however, are also observed in Aycardi et al. (1994), attributed to the "lack of consideration of bond deterioration" in the model. Referring to the exterior joint subassemblage comparisons, the ability to capture degradation and pinching behavior is still noticeably better in the *joint model*. For the comparison in the exterior subassemblage, the *joint model* more accurately predicts the lateral load response in the inelastic range. In terms of deviations in peak lateral load at each cycle from the analysis relative to the experiment, the *joint model* shows an average of 5% improvement in predictive abilities when compared to the *rigid model* for both the interior and exterior joint. As a result of the validation comparisons, it is observed that the *joint model* is better able to reflect the local component behavior, as intended in its development, and instills greater confidence in the model. However, the relative impacts of differences in adopting the modeling strategies within a full building or for probabilistic performance assessment are yet to be explored.

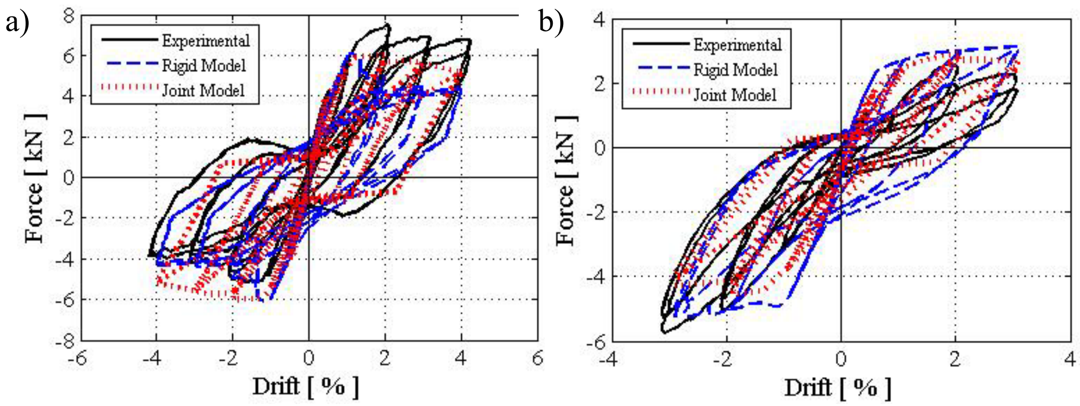

Figure 2: Comparison of the "rigid model" and "joint model" to experimental data for the a) interior subassemblage and b) exterior subassemblage.

ONE-THIRD SCALE MODEL

Bracci et al. (1992a and 1992b) experimentally tested a one-third scale model of a 3-story by 3-bay RC frame. The experimental model was subjected to shaking table tests of the Kern County 1952, Taft Lincoln School Station, N021E component record scaled to three different values of peak ground acceleration (PGA): 0.05g; 0.20g; 0.30g. Table 1 presents a comparison of the natural vibration periods and mode shapes from the finite element models and those derived from the experiment. White noise tests were applied before and after each ground motion record to obtain estimates of the periods and mode shapes of the damaged building in the experiment. The dynamic characteristics of the experimental model, determined from the white noise tests conducted after the scaled ground motion intensities, are stated in Table 1 to compare with the numerical models' dynamic characteristics. The mode shapes are relatively consistent between the models and experiment, whereas some differences exist in the estimates of the periods. During the 0.20g PGA motion the experimental model first yields and begins an inelastic response for the duration of the experiment. The natural periods determined beyond the yielding of the scaled experimental building provide a closer relationship with the natural periods determined from the non-linear FEM analyses. Comparisons in the dynamic responses were made for all levels of ground motion between the experimental data and numerical models developed. The third story displacement time history for the 0.30g PGA motion is shown in Figure 3, highlighting a comparison between the experimental, the *rigid model* and *joint-shear model* responses. The *shear model* response is equal to that of the *rigid model* while the *joint model* response is equal to that of the *joint-shear model* since shear failure was not observed during the analysis, consistent with the experimental test observations. The results also show that there is some deviation in the amplitude of responses throughout the time history, but that the frequency of response is fairly consistent. Furthermore, the peak values of floor displacements are well captured by both models for all levels of ground motion considered, which is of primary interest in most probabilistic seismic response analyses and reliability studies. As shown in Figure 3, there is a 31% and 20% difference in the numerical and experimental results for the *rigid* and the *joint-shear* model, respectively. This error is measured from the maximum displacement along the time history analysis of the ground motion response. This suggests that the *joint-shear* model has a slight advantage in capturing global responses, which is attributed to its ability to capture joint deformations.

Table 1: Dynamic characteristics of experimental vs. analytical modeling techniques; period values in parentheses are the natural periods observed in the experimental model after the Taft 0.05g, 0.20g, and 0.30g PGA ground motions.

Model	**Natural Periods (s)**	**Mode Shapes**		
Experimental	0.97 (1.01) (1.22) (1.44)	1.00	-0.82	-0.46
	0.33 (0.34) (0.40) (0.46)	0.80	0.46	1.00
	0.22 (0.23) (0.28) (0.33)	0.42	1.00	-0.83
"Rigid"	1.30	1.00	-0.81	-0.44
	0.45	0.79	0.50	1.00
	0.30	0.41	1.00	-0.85
"Joint-Shear"	1.34	1.00	-0.81	-0.43
	0.46	0.78	0.52	1.00
	0.30	0.40	1.00	-0.87

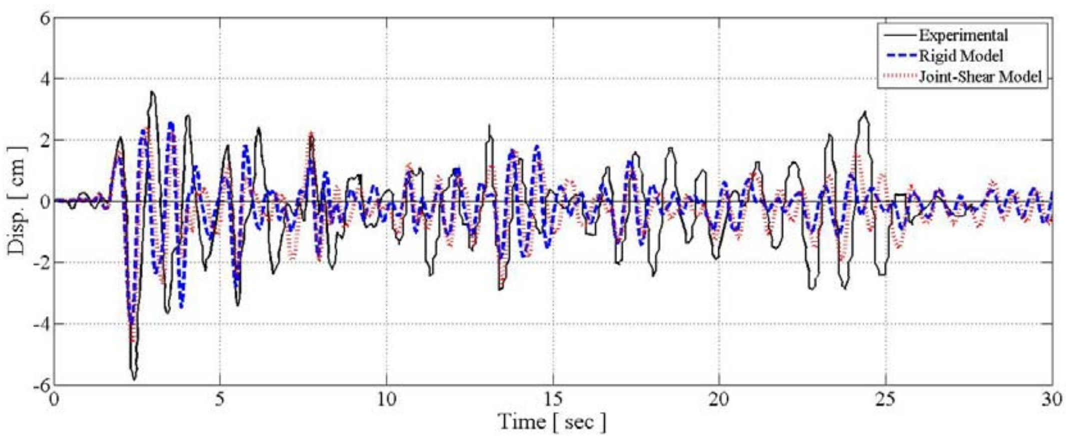

Figure 3: 3rd story displacement comparing one-third scale experimental data with "rigid model" and "joint-shear model" using the Taft motion at PGA of 0.30g.

The results of the comparison with experimental test data suggest that overall the numerical models perform well in capturing component level and global behavior, forming a solid basis for utilization of the models in a probabilistic performance assessment that explores alternative levels of ground motion and inferences regarding form of predictive demand models. They also suggest that the *joint* or *joint-shear* model, as the most rigorous modeling techniques, can provide more accurate estimates of local and global behavior and hence serve as a benchmark comparison in the probabilistic analysis. Since the experimental data for the case study building does not enable explicit validation of the shear model, given that such failures were not observed in the tests and logically not predicted in the simulations, additional comparative studies were conducted to instill confidence in the shear modeling techniques applied to the case study building.

IMPACT OF MODELING APPROACH ON PROBABILISTIC SEISMIC RESPONSE ASSESSMENT

Nonlinear dynamic analyses were performed on the four numerical models described, gathering EDPs in order to make comparisons between local and global demands on the simulated models. Regression models were used as primary methods of comparison between different FEMs for similar EDPs. An uncertainty value is defined from the difference observed in the dispersions from the four numerical models. These methods provide a clear identification and understanding of the discrepancies seen when using alternative modeling techniques.

PROBABILISTIC SEISMIC DEMAND ANALYSIS OF SYSTEM AND COMPONENTS

EDPs were recorded for each model from a "cloud" nonlinear dynamic analysis (Mackie and Stojadinovic 2005). A suite of 220 ground motions was employed for each finite element model discussed. From the 220 ground motions, 100 were taken from a suite created by Krawinkler et al. (2003) and 120 from Baker et al. (2011). These two suites were utilized in order to gather data from a wide spectrum of earthquake characteristics. The types of EDPs gathered from the analysis are seen in Table 2.

Table 2: Local, intermediate, and global engineering demand parameters considered for probabilistic seismic demand models

EDP	Description	Failure Type
Local		
$\varepsilon_{s\ col}$	Maximum column steel strain	Flexural and Axial
$\varepsilon_{c\ col}$	Maximum column concrete strain at extreme fiber	Flexural and Axial
$\varepsilon_{s\ beam}$	Maximum beam steel strain	Flexural
$\varepsilon_{c\ beam}$	Maximum beam concrete strain at extreme fiber	Flexural
$\varepsilon_{j\ int}$	Maximum joint strain at interior column	Joint Shear
$\varepsilon_{j\ ext}$	Maximum joint strain at exterior column	Joint Shear
Intermediate		
V_c	Column shear	Shear Resistance
M_c	Column moment	Bending Resistance
Global		
θ_i	Maximum inter-story drift	Structural and Non-Structural
St. $Acc._i$	Maximum story acceleration	Contents and Non-Structural

The 10 EDPs are defined in terms of local, intermediate, and global characteristics of the building under earthquake loads. Global EDPs are widely accepted as means to assess the overall damage of structures and to provide insight into the damage of non-structural components in buildings. The local and intermediate EDPs chosen are linked to localized component damages observed in past earthquake events for RC buildings and provide sufficient data to make quality comparisons accounting for the effect of the incorporated local models.

A probabilistic seismic demand analysis (PSDA) was conducted using the data recorded in the nonlinear FEM analysis. Cornell et al. (2002) demonstrated that for global EDPs, the seismic demand can be synthetically related to the seismic intensity measure (IM) by using a linear relationship. An IM defines the salient features of the ground motion hazard that affect the structural response. The IM chosen for this study is the spectral displacement ($S_d(T_1)$) corresponding to the fundamental period of the structure. Previous research stated the $S_d(T_1)$ provides efficient correlation for low-story RC buildings with the demands acquired in the analysis.

Finite Element Model Uncertainty

In the PSDA framework, many uncertainty sources have been identified in the past including demand uncertainty (σ_D) (Cornell et al. 2002), modeling uncertainty (σ_M) (Liel et al. 2009), and the capacity uncertainty associated with the performance level (σ_C). The record-to-record variability leads to dispersion in the demand. The modeling uncertainty parameter previously introduced was quantified by considering material property variations. The capacity uncertainty is usually defined by experimentally test for various performance levels. Additionally, modeling uncertainty has been acknowledged due to the use of finite element models to depict real structural behavior (e.g. 2-d representations of real buildings). However, the difference in responses observed due to level of fidelity of different numerical models has yet to be explored.

A new uncertainty parameter (σ_{FEM}) is introduced that focuses on the variation of dispersions from the wide variety of possible FEM techniques. The σ_{FEM} uncertainty is directed at the variation of dispersions in one FEM analysis versus other FEM analyses, whereas other uncertainty parameters primarily focus on variations of objects outside the realm of FEM analysis such as material properties, ground motions, and component capacity. All past researchers make conclusions based on their respective models created as FEMs. There does not seem to be a universal choice of modeling, so σ_{FEM} is needed to incorporate a FEM uncertainty in the PSDA framework. Using the *joint-shear model* as a base comparison, σ_{FEM} is calculated using

$$\sigma_{FEM} = \sigma_{D,i} - \sigma_{D,joint-shear}$$

where $\sigma_{D,i}$ is the dispersion of the demand of the i^{th} model *rigid*, *joint*, and *shear* and $\sigma_{D,Joint-Shear}$ is the dispersion of the demand of the *joint-shear model*. This *joint-shear model* is adopted as a basis of comparison given its superior predictive abilities relative to experimental test data. However, it is acknowledged that an additional source of model uncertainty is still present beyond σ_{FEM}. This parameter is intended to reflect uncertainty introduced by adopting simplified models relative to the best case or most rigorous numerical modeling approach.

From the different models, the σ_{FEM} is calculated for each EDP and averaged to calculate a single FEM dispersion value of 0.12. The individual EDP σ_{FEM} values range from 0.01 for maximum 3rd story acceleration to 0.35 for maximum exterior joint strain. Also, the global EDP σ_{FEM} values are 0.02 and 0.01 for maximum 3rd story drift and maximum 3rd story acceleration, respectively. These low values for global EDPs suggest that added component models minimally affect the PSDA associated with global responses. The average σ_{FEM} value of 0.12 may be added to the dispersion in the PSDA framework for further analysis in fragility analysis, where the total dispersion (σ_T) is

$$\sigma_T^{\ 2} = \sigma_{FEM}^{\ 2} + \sigma_M^{\ 2} + \sigma_D^{\ 2} + \sigma_C^{\ 2}.$$

CONCLUSIONS

Alternative numerical modeling processes are becoming increasingly prevalent in research and industry to gather response data for structures subjected to seismic loading and support probabilistic analysis. In this paper, a non-ductile RC building was modeled based on the design of an experimental setup to compare and validate the results. Joint and shear component models were introduced to the finite element model to observe whether these added components may provide a better approximation to the experimental results. Four combinations were adopted, known as *rigid, joint, shear,* and *joint-shear models.* After performing lateral load analysis to interior and exterior subassemblages and applying earthquake ground motions to the full scale numerical models, the models with the incorporated *joint model* proved to be most accurate as compared to the experimental results. For the subassemblages, there was a 5% improvement in predictive abilities for the *joint models* as compared to the *rigid model*. Also, the *joint models* have more accurate representations of pinching and strength degradation in the force-deformation response of the subassemblages. The peak deformation responses of the time history displayed differences of 31% for *rigid model* as compared to 20% for the *joint-shear model.* Hence, the addition of the component models in the subassemblage and full scale models provide improved approximations relative to the experimental data. The components models are also able to record intermediate and local engineering demand parameters.

A new finite element modeling uncertainty parameter (σ_{FEM}) was determined from probabilistic seismic demand analyses. The σ_{FEM} arises from the difference of dispersions observed between the four different numerical models used to record demands for the probabilistic assessment. This quantity reflects the uncertainty introduced by using more simplified modeling techniques. The overall average σ_{FEM} is found to be 0.12 across the local and global EDPs for all models relative to the *joint-shear model*. This new uncertainty parameter helps establish a relationship between the various levels of fidelity of FEM techniques used in research and industry. Its incorporation in PSDA can better reflect the uncertainty inherent in vulnerability modeling of RC structures conducted by using simplified FEM approaches. Future studies should explore the range in σ_{FEM} observed across different geometries beyond the case study, level of variation among global and local EDPs, and relative contribution of σ_{FEM} to other sources of uncertainty in seismic risk assessment and loss estimation.

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